Multi-Unit Auctions with Dependent Valuations: Issues of Efficiency in Electricity Auctions

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Abstract

As auction based mechanisms for electricity dispatch are emerging in previously regulated electricity supply industries, it is imperative to understand the effect of auction rules and structure on efficiency. This paper addresses exactly this relationship in a complete information framework by asking which auction structures are sufficient to guarantee that electricity demand is satisfied in a least-cost manner.

1 Introduction

Many governments are finding it in their and their constituents' best interest to deregulate their electricity supply industries. The goal of the regulators is to introduce competition into their electricity supply industries and create the appropriate medium through which electricity buyers and sellers can actively trade electricity, in the hope that such a competitive market will promote efficiency.

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As of January 1, 1998, California will join the ranks of deregulated electricity supply industries. At the center of California's deregulation effort is an electricity auction mechanism, the Power Exchange (PX), whose role is to facilitate the matching of supply with all or part of their electricity demand. Auction-based mechanisms for electricity dispatch have already been implemented in the United Kingdom and Australia. Each of these governments has chosen an auction mechanism which in this paper is identified as a uniform price, vertical, simultaneous auction. The design of an electricity auction which induces an efficient use of generation resources is complicated by the fact that electricity demand, which fluctuates from hour to hour, must be satisfied by many suppliers with different costs, and that the generators lack the ability to store electricity in inventory. Determination of the optimal dispatch is a computationally difficult problem even in a centralized model with known generator costs. It is an even greater challenge to design an auction where generators voluntarily reveal cost information so as to be efficiently dispatched.

The purpose of this paper is to analyze the performance of different auction structures in minimizing generation costs. In particular, I am interested in identifying which, if any, rules in an auction structure are necessary and sufficient to guarantee productive efficiency, i.e., minimization of generation costs, in equilibrium. I focus only on the generation(supply) side of the market and assume that the demand for electricity is both deterministic and inelastic. This is done in order to gain a better understanding of the incentives provided by different auction structures and the ability of each auction structure to induce the efficient[1] allocation, or *efficient dispatch*. Under these assumptions, the final allocation is efficient if the generators chosen to supply electricity (i.e., win dispatch) minimize *total* generation costs.

I find that the auction mechanism chosen in the United Kingdom, Australia and California cannot guarantee productive efficiency in equilibrium. The main failing of a uniform price, vertical, simultaneous auction is the way demand is bundled and hence the way bids are defined. I show that, while an auction mechanism which allows for more than one winner per lot (to be defined in section 2.2) cannot guarantee efficiency, an auction when there is exactly one winner per lot can guarantee productive efficiency.

In section 2, I provide the reader with some background literature on multi-unit auctions. I then

[1] I will assume that there exists a unique efficient dispatch. This assumption does not in any way alter the results of the paper.

go on to characterize an electricity auction as a multi-unit auction with private valuations which are possibly dependent over several units, and outline the different auction mechanisms considered and the cost characteristics of generators. In section 3, I argue that bundling demand into lots which allow for more than one winner precludes guaranteeing the efficient dispatch in equilibrium. In section 4, I present an auction mechanism that does guarantee efficiency in equilibrium.

2 Electricity Auctions

2.1 Auction Literature

In order to appreciate the new and interesting questions posed by an electricity auction, it is important to examine its place in the existing auction literature. The largest portion of auction literature looks at models where bidders desire at most one object (McAfee and McMillan (1987) provide an excellent survey of the auction literature). As I shall explain in more detail in the next section, an electricity auction is a multi-unit demand auction with private, dependent valuations.[2] Hence it is not possible to apply the results from single unit auctions to an electricity auction. Studying auctions for electricity forces us to leave the well-studied and understood world of private and constant valuations and explore the performance of auctions in a more diverse setting.

Another area which is spurring greater interest and research in multi-unit auctions with dependent valuations is the recent FCC spectrum auctions. In the FCC auctions, bidders, comprised of US telecommunication companies, cellular telephone companies, and cable-television companies, competed to win various spectrum licenses for different geographical areas. The synergies arising from owning licenses in adjoining geographical area creates dependencies in (some) bidders' valuations for individual licenses. See McMillan (1994), Cramton (1995) and McMillan and McAfee (1996) for further discussion of the FCC spectrum auctions.

Several papers have addressed the issue of multi-unit auctions. Wilson (1979) began the study of "share" auctions, where bidders with a common valuation submit demand curves and are awarded a fraction of the shares at a market-clearing price. Maskin and Riley (1989) study the design of

[2]The dependency in valuations is not across bidders, as assumed in a common valuations model, but instead refers to one bidder's dependent valuations over many objects. In this paper, the objects for auction are 1 MWh of energy.

optimal multi-unit auctions with private valuations. Hausch (1986) studies a two object auction, where there are two bidders with common valuations who desire both objects. Hausch finds that the seller's revenue is greater when both objects are sold simultaneously versus sequentially. Gandal (1997) looks at the sequential auctioning of cable television licenses in Israel, where there may exist some interdependencies among licenses' valuation. Krishna and Rosenthal (1996) study auctions where there are two types of bidders, global and local. Global bidders desire more than one object and their valuation for multiple objects is greater than the sum of each individual object's valuation, while local bidders desire at most one object. Krishna (1993) uses a complete information framework (as assumed in this paper) to analyze the optimal strategy for a monopolist and competitive fringe when additional capacity is made available for purchase over time. Bikchandani (1996) establishes the relationship between the existence of allocatively efficient Nash Equilibrium for 1^{st} and 2^{nd} price auctions of individual heterogeneous objects and Walrasian equilibrium under complete information. Rothkopf, Pekec, and Harstad (1995) identify a special class of multi-unit auctions in which bidders can submit a bid for different combinations of objects and the auction is computationally manageable. Ausubel and Cramton (1996), using Wilson's "share" auction framework with private valuations, establish that the efficiency of 2^{nd} price (uniform) auctions in a single-unit auction do not carry over to a multi-unit framework. They conclude that when bidders desire more than one object, they have an incentive to underbid or "shade" their bids resulting in an inefficient allocation.

Von der Fehr and Harbord's (1993) analysis of the UK Electricity Industry is the only other study I know of that identifies an electricity auction as a multi-unit auction with private valuations and attempts to study the strategic bidding behavior of generators. Von der Fehr and Harbord assume a complete information framework where demand is uncertain but its distribution is known, with two generators who have (different) constant marginal costs of generation. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium. Building on their analysis, I incorporate the presence of fixed "start-up" costs into generation costs and extend their study of bidding behavior to alternative auction structures.

2.1.1 Multi-Unit Dependent Valuations Auction

What separates designing an auction for electricity from the vast body of auction literature is the structure of generation costs. Generators have different types of costs (e.g., ramp-up costs, no-load costs, etc.) which must be recovered through their sales revenues. Generation costs can be broadly classified into two groups: fixed "start-up" costs are incurred when a generator is turned on to generate, and variable costs are incurred with each additional MWh generated. Due to this cost structure, there exist cost *dependencies* in both time and quantity dimensions, i.e., the (average) cost to generate 1 MW during hour t depends upon the number of additional MW generated during hour t and other hours.

If the object being auctioned is defined as 1 MWh of demand for energy, then demand can be interpreted as a collection of identical objects, distinguished only by the time at which they occur. Generators are sellers of electricity who may wish to win many objects (by winning an object they win the right to supply that 1 MWh of demand at a price determined through the auction process) and hence have *multi-unit demand*.

A generator's profit from "winning" a MWh is the difference between the auction price it is paid and its own *private* cost for supplying the MWh. A generator is constrained from winning all objects (and hence supplying the entire demand the following day) by the presence of capacity constraints on its generation level at any point in time (i.e., if a generator has a capacity of K, the maximum level of MW at which it can generate at any point in time is K MW). For most hours in the day, demand is greater than the capacity of any one generator; hence several generators must be chosen to supply demand.

Hence, an electricity auction is a multi-unit auction where there exist dependent private valuations for the units and "purchasing" capacity constraints.

2.2 Auction Structures

In designing an electricity auction, the auctioneer must decide on such auction dimensions as: how to bundle demand into lots for auction, what the pricing rule will be and what the sequencing of the auction will be. The United Kingdom, Australia and California have decided to bundle demand into vertical lots (explained below), to pay the same uniform price to every winning generator in a lot, and to have generators submit their bids for all lots simultaneously. As will be shown in section 3.1, this auction structure does not guarantee efficiency in equilibrium. In order to understand why this is true and how the auction might be modified to remedy this problem, I identify the different auction dimensions and the possible alternatives within each dimension.

Bundling of Demand into Lots In the case of electricity, the basic object to be auctioned is 1 MWh of the forecasted daily demand. When there are several objects to be auctioned, the auctioneer must decide how to bundle the objects into lots for auction. Should the bidders submit one bid that applies equally to all objects, or should the object be divided into distinct lots for which bidders submit a separate bid for each lot?

In the context of an electricity auction, there exist two "natural" bundling forms: horizontal and vertical.

Definition 1 *In a* **horizontal auction** *(see Figure 1), demand lots are formed by partitioning daily demand according to its duration, i.e., a distinct lot for each duration t. Hence, generators submit a supply curve for each lot, indicating the price at which they are willing to generate k megawatts for a* **duration** *of t hours, where $k, t > 0$.*

Definition 2 *In a* **vertical auction** *(see Figure 2), daily demand is divided into T hourly demand lots, where each demand lot contains all the demand in hour t, $t = 1...T$. For each hour t, generators bid the prices at which they are willing to generate k megawatts* **during** *hour t, where $k, t > 0$.*

While there exist countless ways to bundle demand, the bundle forms identified here are the most practical and logical in an electricity auction setting. As stated earlier, the electricity auctions in operation in the United Kingdom and Australia and the proposed auction for California are vertical auctions. For the purpose of planning their generators' schedules (or dispatch), however, individual generation companies interpret demand in horizontal slices and use an integer-program to solve for the least-cost dispatch.

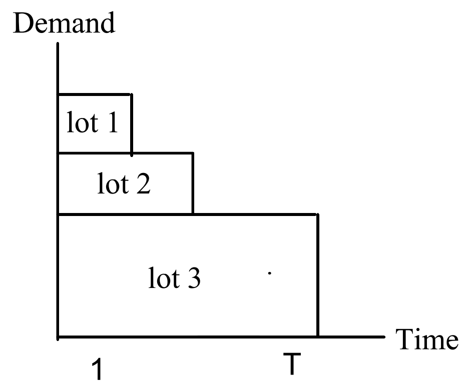

Figure 1: Horizontal auction

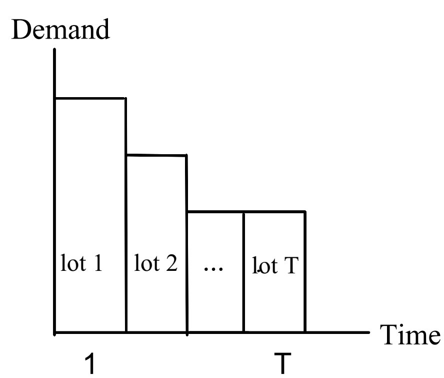

Figure 2: Vertical auction

Sequencing of Auctions When there is more than one demand lot to be auctioned, the auctioneer must decide how to sequence their sale. In a simultaneous auction, the bids are submitted, and allocation decisions for all demand lots are made simultaneously. Alternatively, in a sequential auction, demand lots are auctioned sequentially; before each new auction, the results of any previous auctions are made known.

Pricing Rule Before the bidders submit their bids, they must know how the prices at which the transactions take place are to be determined. If there is more than one winner per lot, each with different bids, in a lot, do the winners pay the same price, or different prices? The former pricing rule is called a *uniform pricing rule* and the latter a *discriminatory pricing rule*. Under a uniform pricing rule, all winners in a lot are paid the highest accepted bid price.[3] Under a discriminatory pricing rule, each winner is paid its own bid price.[4]

Figure 3 lists all the possible auction mechanisms given the dimensions and choices identified above.

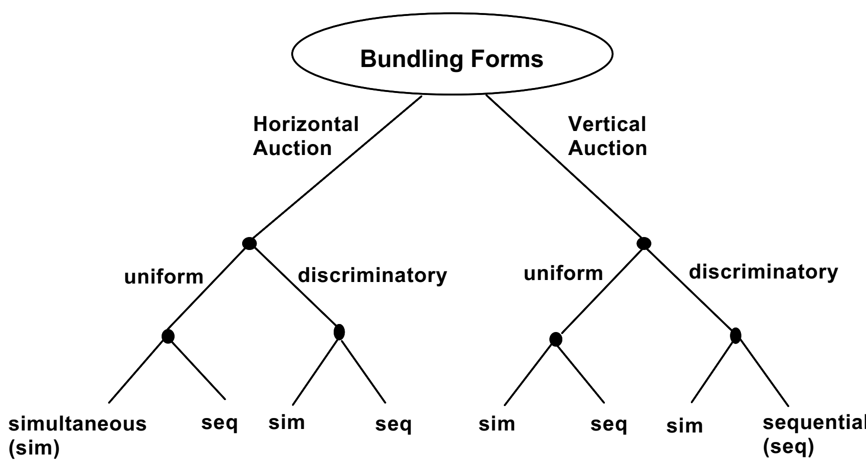

Figure 3: Possible auction mechanisms.

[3]This is because an electricity auction is a procurement auction. In a procurement auciton, the goal is solicit bids from suppliers for a service.

[4]Both auction pricing rules implicitly determine the transaction price(s) using a 1^{st} price rule.

2.2.1 Model

In order to gain a better understanding of the incentives provided by different auction structures and the ability of each auction structure to induce the efficient dispatch, I assume that electricity demand is forecasted, i.e., is both deterministic and inelastic. For simplicity, forecasted demand is always assumed to be a step function which is constant during each hour [5], and is public information and hence known to all generators.

Throughout the paper I will assume that a generating plant can be one of n technology types. Assume that there exist nM generating companies (referred to as generators) who each own a finite number of identical generating plants (see Figure 4). A generator who owns plants of technology type i is denoted by G_{ij}, $i = 1...n$, $j = 1...M$. Each plant has two costs associated with generation: a fixed "start-up" cost, f, and a variable cost per MWh, v, once the plant is up and running. The cost of generating a total of Q MWh in T hours from a type i plant is $C_i(Q) = f_i + Qv_i$,[6] where $Q = \sum_{t=1}^{T} q_t$ where q_t is the number of MW generated during hour t and $q_t \leq K$. The upper limit on the value of q_t, K, is a plant's capacity. The capacity constraint implies that a plant cannot supply more than K MW at any point in time, but places no restrictions on the duration for which it can generate. Assume that all generation plants have the same capacity constraint of K MW.

Figure 5 plots the total costs of generation associated with each type of plant technology, assuming a generating plant is "switched on" only once (The horizontal axis measures the total number of MWh generated over time. Numerical examples of fixed and variable costs have been provided for convenience of the reader). Note that costs are such that each type i is the least-cost technology for some output level, in this case, i MWh. This is done so as to correctly reflect reality: most thermal generation plants are either nuclear, coal or gas-fired, and each fuel type is the most efficient source over some output range.

Assume that all cost and capacity information is publicly known. The assumption of complete information may at first seem to be quite restrictive and unrealistic. However, in an industry such as the electricity supply industry, which has a long history of regulation (often with all the generation sources under government management as was the case in the United Kingdom before 1990), it is

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[5] Any daily demand can be approximated, to a 1^{st} order, by a step-function.

[6] A generator incurs no cost if it is not turned on to generate, i.e., $C_i(0) = 0$.

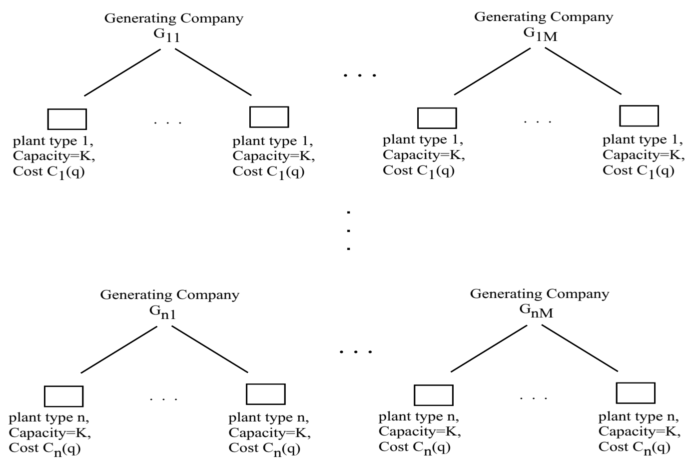

Figure 4: Each generator owns several identical generation plants.

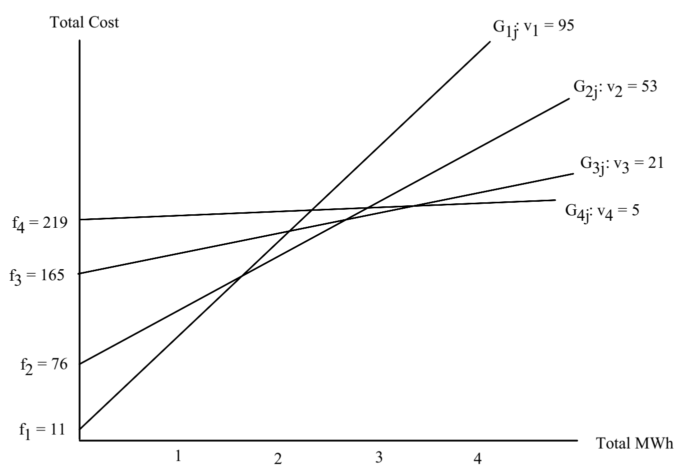

Figure 5: Total cost for plant technology types $n = 1, ..., 4$.

quite reasonable to assume that generators are aware of what types of plants their competitors own and what the costs associated with generation are. As a first attempt to understand the relative strengths and weaknesses of different auctions mechanisms, it is quite informative to examine generators' behavior in an environment with complete information.

When submitting bids, generators are restricted to submitting a single, uni-dimensional, integer bid per demand lot per generating plant.[7] A generator's bid is binding once it has been submitted. For a vertical auction, the bid indicates the minimum price to generate 1 MW during hour t; for a horizontal auction it indicates the minimum price to generate 1 MW for t hours. It is important to point out that the results in this paper also hold when generators are allowed to submit a step bid function per generating plant. (See Appendix A for further discussion.) Generators are dispatched in increasing order of bids for a lot. A "winning" generator in a lot is dispatched at its full capacity or until demand in the lot is exhausted, i.e., a generator is not able to restrict its capacity availability.

3 Demand Lots with Many Winners

The question of interest to us is: In a complete information setting, does the proposed auction induce profit-maximizing bidders to bid in a way that always results in an efficient dispatch in Nash equilibrium[8], i.e., are all equilibrium efficient for all demand scenarios? I find that the answer is negative. The demand bundling forms under consideration, horizontal and vertical, are such that there can be more than one winner in a lot. This, in turn, creates incentives for generators to not bid their true costs and can preclude attaining the efficient dispatch.

Theorem 1 *In a complete information setting with strictly concave generation costs, none of the auction mechanisms in Figure 3 can guarantee the efficient dispatch in equilibrium.*

The proof to Theorem 1 is presented in this section via counterexamples. I address the failings of all forms of vertical and horizontal auctions in sections 3.1 and 3.2, respectively. For each of the

[7] In reality, generators may submit a separate bid per genset. For simplification, this paper assumes that a genset and plant are identical.

[8] Or a subgame perfect Nash Equilibrium in the case of a sequential auction.

auction mechanisms in Figure 3, I am able to find an instance of demand for which either 1) there exists an inefficient equilibrium or 2) the efficient dispatch is not supported in equilibrium. Hence, none of the auction mechanisms can guarantee efficiency in equilibrium. While the examples in this paper will assume a cost function of the form "start-up" plus constant variable cost and a setting with three generators, the results hold for any strictly concave cost function and any number of generators (greater than 3).

Assume throughout section 3 that there are four technology types, i.e., $n = 4$, and each generator owns one plant with a capacity of $K = 2$ MW. In addition, suppose that forecasted demand is as in Figure 6. This demand model, albeit a simple one, is rich enough to illustrate the failings of all the auctions in Figure 3.

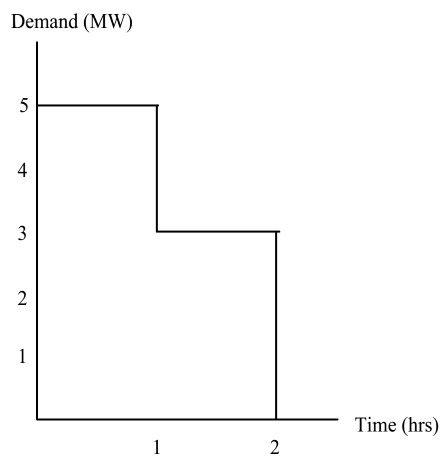

Figure 6: Daily demand

Given the assumed demand, cost, and capacity functions, the unique efficient dispatch is given in Figure 7.[9] The efficient dispatch consists of the same type 4 generator (G_{4j}) supplying 2 MWh in both time periods, the same type 3 generator (G_{3j}) supplying 2 MWh in the 1^{st} period and 1 MWh in the second, and a type 1 generator (G_{1j}) supplying the top 1 MW. For notational simplicity and without loss of generality, assume that the winning generators in the efficient dispatch are G_{11}, G_{31}, and G_{41}.

[9]The efficient dispatch is always the same, regardless of the auction mechanism.

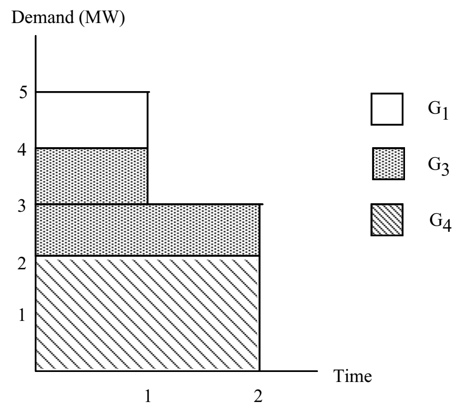

Figure 7: Efficient dispatch.

3.1 Vertical Auction

A vertical auction of the demand in Figure 6 consists of the auction of two lots, as in Figure 8. Each generator G_{ij} submits two bids, one for each lot k, defined to be b_{ij}^k (recall that each generator owns only one plant and therefore only submits one bid per lot). A bid of b_{ij}^k is the minimum amount generator G_{ij} must be paid to generator 1 MWh in lot k. In order for the efficient dispatch to result from the submitted bids, G_{11}, G_{31}, and G_{41} must submit the lowest bids in *lot 1* and G_{31} and G_{41} must submit the lowest bids in *lot 2*, regardless of the pricing rule or auction sequencing. In lots 1 and 2, the winning bids must be ordered as follows: $b_{41}^1 < b_{11}^1$, $b_{31}^1 < b_{11}^1$, and $b_{41}^2 < b_{31}^2$.

Vertical Uniform Auction In a uniform price auction, all generators who "win" and are dispatched in a given demand lot are paid a uniform price equal to the highest accepted bid. When there is more than one winner per demand lot, a uniform pricing is unable to guarantee an efficient dispatch in equilibrium because: (1) The bid price is separated from received price for all except the marginally dispatched generators, (2) The same $/MWh price is paid for each MWh generated in a demand lot, and (3) The total cost curve is strictly concavity.

Given the demand in Figure 6 and capacity assumptions, Table 1 defines a set of equilibrium bids for generators G_{ij}, $i = 1...4$, $j = 1...M$, which constitute an *inefficient* dispatch for a vertical, uniform, simultaneous and a vertical, uniform, sequential auction (see Figure 9 for the inefficient

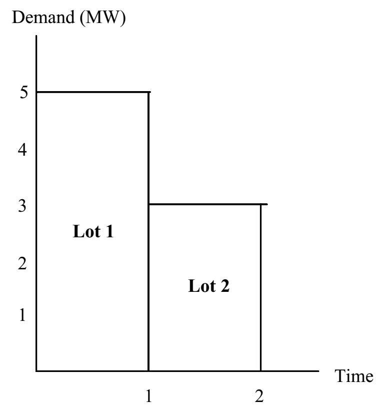

Figure 8: Vertical auction of demand.

dispatch). The winning bids are followed by an asterisk (Note: Neither these bids nor dispatch constitute the unique equilibrium for either auction).

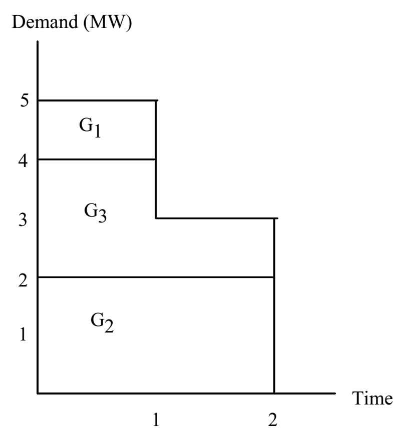

Figure 9: Inefficient equilibrium dispatch.

Given their opponents' strategies in Table 1, no generator has an incentive to deviate from its bids. This profile of bids results in G_{21} being dispatched for 2 MWh in *lots 1* and *2*, G_{31} being dispatched for 2 MWh in *lot 1* and 1 MWh in *lot 2*, and G_{11} being dispatched for 1 MWh in *lot 1*. The clearing price received by all winning generators in *lot 1* is $f_1 + v_1$ per MWh and in *lot 2* is $v_1 - 1$ per MWh. Since G_{21}'s payoff is not determined by her bid, she has every incentive to bid

Generator	Bid for Lot 1	Bid for Lot 2
$\{s = 2...M, y = 1...M\}$	b_{ij}^1	b_{ij}^2
G_{11}	$f_1 + v_1^*$	v_1
G_{1s}	$f_1 + v_1 + 1$	$f_1 + v_1$
G_{21}	0^*	0^*
G_{2s}	$f_2 + v_2$	$f_2 + v_2$
G_{31}	0^*	$v_1 - 1^*$
G_{3s}	$f_3 + v_3$	$f_3 + v_3$
G_{4y}	$f_4 + v_4$	$f_4 + v_4$

Table 1: Generator bids for lots 1 and 2 in a vertical, uniform auction

as low as possible to ensure dispatch. By submitting a bid of zero in lots 1 and 2, G_{21} is able to win dispatch at a positive profit. Although G_{41} is (one of) the least-cost producers of 4 MWh, he is unable to profitably undercut G_{21}'s bids of $b_{21}^1 = b_{21}^2 = 0$.

This simple example clearly illustrates why a vertical, uniform auction cannot guarantee efficiency in a multi-unit environment with strictly concave costs. If the demand in any hour t is not an integer multiple of K, then not all the generators will be dispatched at the same output level within that hour. In this scenario, there exists an opportunity for a relatively inefficient generator to accrue a positive profit by bidding zero and ensuring dispatch without fear of receiving its below-cost bid price. With the knowledge that in equilibrium the clearing price is guaranteed to be at least the cost of the marginal bidder in hour t, a relatively inefficient generator can "sneak-in" to the dispatch schedule by submitting a zero bid, get dispatched at a higher level in hour t than the marginal price-setting bidder and accrue a positive profit due to the concavity of its cost curve. This "zero" bid strategy creates a very similar effect to one identified by Back and Zender (1993) for the uniform auction of Treasury bills. Bidders are able to costlessly deter competitors from bidding more aggressively by submitting extremely steep demand curves. The low bids on inframarginal quantities have no chance of determining the clearing price, but act as a deterrent to competitors from bidding more aggressively.

This result bears directly on the auction mechanisms chosen in the UK, Australia and California.

All three auctions are designed so as to have generators submit hourly (or half-hourly) bids: The winners in each time period are paid the highest accepted price. This is exactly the structure of a vertical, uniform auction and hence we should not expect the auctions to be providing generators with the correct incentives so as to result in the efficient dispatch.

Vertical Discriminatory Auctions While a uniform-price vertical auction of the demand in Figure 6 fails to guarantee efficiency in equilibrium because of the existence of inefficient dispatches in equilibrium, a discriminatory-price vertical auction fails because it cannot support the efficient dispatch in equilibrium. The following exposition is true for both a vertical, discriminatory, sequential and a vertical, discriminatory, simultaneous auction.

In a discriminatory-price auction, each generator chosen for dispatch receives its own bid. For the efficient dispatch to occur in equilibrium in a vertical auction, (as stated earlier) the three lowest bids in *lot 1* must be from a type 4,3, and 1 generator (with the type 1 generator having the third lowest bid) and the two lowest bids in *lot 2* must be from a type 4 and 3 generator (with the type 3 generator having the second lowest bid). As there are many identical generators of type 1, in an efficient equilibrium, G_{11} must be earning zero profits. If G_{11} were earning a positive profit, then any generator G_{1j}, $j \neq 1$, would have the incentive to undercut b_{11}^1 and replace G_{11} as the third lowest bidder in *lot 1*. Therefore we know that, in an efficient equilibrium, G_{11} will bid $b_{11}^1 = f_1 + v_1$. Since each generator is paid what it bids, and each generator wishes to maximize its profits, in an efficient equilibrium, G_{31} and G_{41} will bid $b_{31}^1 = b_{41}^1 = f_1 + v_1 - 1$, and all other generators must submit bids for *lot 1* greater than $f_1 + v_1$. At a bid of $b_{31}^1 = f_1 + v_1 - 1$, G_{31} is dispatched at 2 MWh in *lot 1* and earns $2b_{31}^1 > f_3 + 2v_3$. There are, however, many other type 3 generators who are not being dispatched and are earning zero profits. Therefore, any G_{3j} , $j = 2...M$, has the incentive to undercut b_{31}^1 and replace G_{31}'s position in the bid ordering. But then we have just shown that the bids (and bid ordering) necessary to support an efficient dispatch are not equilibrium bidding strategies. Therefore, the efficient dispatch cannot be supported in a discriminatory-price, vertical, simultaneous or sequential auction.

The structure of vertical demand lots creates the need for more than one winner per lot, which in turn creates a barrier to guaranteeing efficiency. A generator's dispatch depends upon its bids' placement within in a lot. In this example, the efficient dispatch requires that different types of generators, with different cost structures, win within a lot. This fact, coupled with the winning generators' desire to maximize their profits, creates a situation where G_{31} and G_{41} bid above their costs. But in the presence of other identical generators, it cannot be an equilibrium for G_{31} and G_{41} to bid above their cost without other generators undercutting their bids.

Given the concavity of total costs, it is not possible to design a vertical auction which is able to provide generators with the proper incentives so as to guarantee the efficient dispatch in equilibrium.

3.2 Horizontal Auction

A horizontal auction of the demand in Figure 6 consists of auctioning two lots as in Figure 10. Each generator G_{ij} submits two bids, one for each lot k, defined to be b_{ij}^k (recall that each generator owns only one plant and therefore only submits one bid per lot). A bid of b_{ij}^k is the minimum amount generator G_{ij} must be paid to generator 1 MW for k time periods. In order for the efficient dispatch to result from the submitted bids, G_{31} and G_{41} must submit the lowest bids in *lot 2* and G_{31} and G_{41} must submit the lowest bids in *lot 1*, regardless of the pricing rule or auction sequencing. In lots 1 and 2, the winning bids must be ordered as follows: $b_{41}^2 < b_{31}^2$ and $b_{31}^1 < b_{11}^1$.

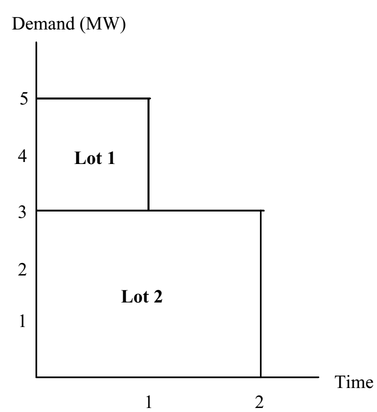

Figure 10: Horizontal auction of demand.

Horizontal Uniform Auction The inefficiencies associated with a uniform price are not limited to vertical auctions. In a horizontal auction, demand is divided into lots by duration. There

Generators	**Bid for Lot 2**	**Bid for Lot 1**
$s = 2...M,\ y = 1...M$	b_{ij}^2	b_{ij}^1
G_{1y}	$f_1 + 2v_1$	$f_1 + v_1^*$
G_{21}	0^*	$f_2 + v_2$
G_{2s}	$f_2 + 2v_2$	$f_2 + v_2$
G_{31}	$f_3 + 3v_3 - (f_1 + v_1)^*$	0^*
G_{3s}	$f_3 + 3v_3 - (f_1 + v_1) + 1$	$f_3 + v_3$
G_{4y}	$f_4 + 2v_4$	$f_4 + v_4$

Table 2: Bids which yield inefficient dispatch in a horizontal uniform auction

is no reason to believe that the number of MW of demand with duration t will equal a multiple of the generators' capacity, i.e., it is possible to have more than one generator win dispatch in a given demand lot at different output levels. In such a scenario, a strategy similar to the one used in a vertical uniform auction can lead to an inefficient dispatch being supported in equilibrium, i.e., a relatively inefficient generator can ensure its dispatch by submitting a low-bid, while reaping a positive profit from the marginal price.[10] This section presents an example of an inefficient equilibrium in a uniform, horizontal auction. The strategies presented constitute an inefficient equilibrium for both a sequential and a simultaneous uniform, horizontal auction.

Given the demand in Figure 6 and capacity assumptions, Table 2 defines a set of equilibrium bids for generators G_{ij}, $i = 1...4$, $j = 1...M$, which constitute an *inefficient* dispatch in a horizontal, uniform, simultaneous and a horizontal, uniform, sequential auction (see Figure 9 for the inefficient dispatch). The winning bids are followed by an asterisk. Note: the type one winner, G_{1y}, will be chosen randomly.

Given her opponents' bids for *lot 2*, G_{21} has the incentive to bid low so as to ensure dispatch. By submitting a bid of $b_{21}^2 = 0$, G_{21} wins a dispatch of 2 MW for 2 hours and wards off her opponents' from undercutting her bid. If any other generator, in particular the efficient generators G_{4j}, were to match G_2's bid and submit a zero bid for *lot 2*, the clearing price would be zero and the dispatched generators would generate at prices below their costs. Since the demand in *lot 2*

[10]This strategy would not be profitable in equilibrium if all winning generators were dispatched the same amount in each demand lot.

is greater than the capacity of G_{21}, G_{31} also wins 1 MW dispatch and sets the clearing price at $f_3 + 3v_3 - (f_1 + v_1)$. Due to the height of *lot 2* of 3 MW, G_{21} is dispatched at twice G_{31}'s level and receives twice the payment. Hence, the uniformity of bids and unequal dispatch of generators in a demand lot combined with the concavity of its cost function allow G_{21} to recoup its generation costs with a bid of zero and inefficient dispatch to be supported in equilibrium.

Horizontal Discriminatory Auction Given the demand in Figure 6, in order for the efficient dispatch to occur from the submitted bids in a horizontal discriminatory auction, the lowest and second lowest bid in *lot 2* must be from G_{41} and G_{31}, respectively, and G_{31} and G_{11} must be lower than all other generators in *lot 1*. As was shown in the case of a vertical, discriminatory auction, the need for more than one type of generator to win within a lot combined with generators' profit maximizing behavior bars the efficient dispatch from being attainable in equilibrium. The following exposition is true for both a horizontal, discriminatory, sequential and a horizontal, discriminatory, simultaneous auction.

The presence of several identical type 1 generators implies that in equilibrium the winning generator G_{11} must accrue zero profit. If he were to bid above his cost of $f_1 + v_1$ in *lot 1,* one of his opponents of the same technology would have an incentive to undercut his bid. Similar reasoning can be used to argue that in an efficient equilibrium dispatch, G_{31} must earn zero profits and hence bid $b^2_{31} = f_3 + 3v_3 - (f_1 + v_1 - 1)$.[11] For the efficient dispatch, the bids in *lot 2* must be such that b^2_{41} is below b^2_{31} and all other generators (including all other type 4 generators) bid above b^2_{31}. Profit maximization will imply that $b^2_{41} = f_3 + 3v_3 - (f_1 + v_1 - 1) - 1$: At such a bid, G_{41} is dispatched for 4 MWh and earns a positive profit. But then it is not an optimal response for all other type 4 generators to bid above b^2_{31}, not be dispatched and earn zero profit. Instead, they have the incentive to undercut b^2_{41} and replace G_{41} as the lowest bidder. Hence, as was the case with the vertical, discriminatory auction, the efficient dispatch cannot be supported in equilibrium.

Thus it has been shown that none of the auctions in Figure 3 can guarantee efficiency in equilibrium.

[11] In order to win dispatch in lot 1 while maximizing profits, G_{31} will bid $f_1 + v_1 - 1$.

4 Demand Lots with One Winner

Section 3 establishes the inability of auction mechanisms that bundle demand into horizontal and vertical lots to guarantee efficiency in equilibrium when there exist strictly concave generation costs. The main flaw with horizontal and vertical lots is the possibility of there being more than one winner per lot. Therefore, it is appropriate to search for an alternative bundling form which assures only one winner per lot. When demand is partitioned vertically, it is not possible to achieve only one winner per lot. However, it is possible with horizontal partitioning.

Definition 3 *In a* **1- horizontal auction** *(see Figure 11), demand lots are formed by partitioning daily demand into 1 MW horizontal strips. Generators submit a bid for each lot, indicating the price at which they are willing to generate* 1 *megawatts for a* **duration** *of t hours, where t is the length of the strip.*

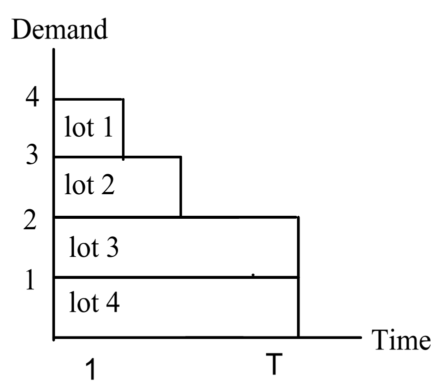

Figure 11: *1-horizontal* auction of demand

By partitioning demand into thin horizontal strips, whose height, of 1 MW, is less than or equal to the capacity of generators, it is possible to limit the number of winners per lot to one. A $1-horizontal$ auction is by definition, a discriminatory auction since there is only one winner per lot. There is still the possibility of conducting the auction of the lots simultaneously or sequentially. I find that while the existence of only one winner per lot is necessary to guarantee efficiency, it is not sufficient. The final characteristic upon which efficiency depends is the sequencing of an auction.

When bids are made sequentially and there are few generators participating in the auctions, a $1 - horizontal$ auction cannot guarantee efficiency. However, when all the bids are submitted simultaneously, the unique Nash Equilibrium dispatch is the efficient dispatch.

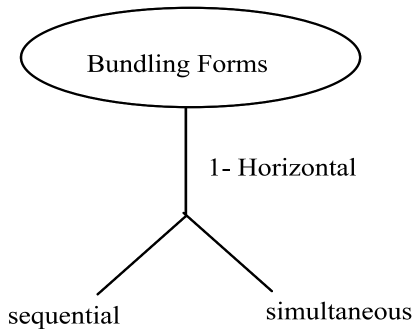

Figure 12: Alternative bundling forms and auction mechanisms.

Section 4.1 contains an example of a sequential auction of demand in which the set of equilibrium dispatches contains an inefficient dispatch. Section 4.2 concludes with a proof stating that the unique equilibrium dispatch in a simultaneous *1 − horizontal* auction is the efficient dispatch.

4.1 1-Horizontal Sequential Auction

A *1 − horizontal* auction cannot guarantee efficiency in equilibrium if there exist few generators participating in the auction when demand lots are auctioned sequentially. With few participants, e.g., one generator of each type $i = 1...n$, relatively inefficient generators are able to strategically bid so as to squeeze out an efficient competitor. The ability to support inefficient dispatches in equilibria, however, critically rests on the number of generators participating in the auction. If there exist many[12] generators of each technology type, then all equilibria are efficient.

Theorem 2 *In a complete information framework with few generators, a*

1-horizontal sequential auction cannot guarantee efficiency in equilibrium.

[12] Where "many" implies that there are more generators of each type than demand for which it is the most efficient technology.

Proof. Theorem 2 is proved by counterexample. Suppose that there are three generators, G_1, G_2, and G_3 who each own two identical generating plants with $K = 1$ each. The generation costs associated with (both of) G_i's plants are given in Figure 13, $i = 1, 2, 3$. Suppose the generation costs are such that $f_2 + 3v_2 - (f_3 + 3v_3) < f_3 + 2v_3 - (f_1 + 2v_1)$.

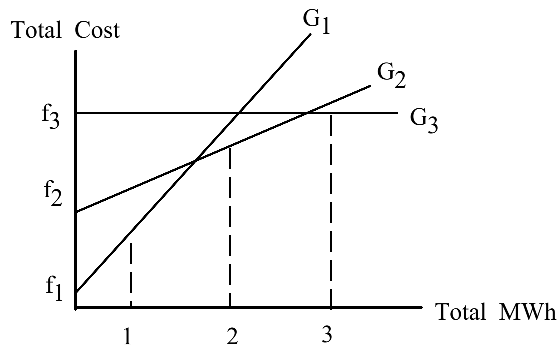

Figure 13: Generation costs per plant for generation G_i, for $i = 1, 2, 3$.

Assume that there exists a daily demand given by Figure 14, which is to be auctioned via a *1 − horizontal*, sequential auction. The longest duration lot, *lot* 3, is auctioned first, followed by *lot* 2 and then *lot* 1. Figure 14 also depicts the unique efficient dispatch. Despite the simple structure of demand, it is possible to support an inefficient dispatch in equilibrium, in particular the dispatch given in Figure 15.

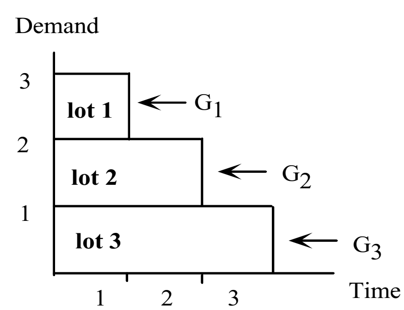

Figure 14: The unique efficient dispatch for a sequential *1-horizontal* auction.

The equilibrium strategies supporting this inefficient dispatch are given in Appendix B. I briefly summarize here the strategies used and incentives behind these strategies. Recall that each gener-

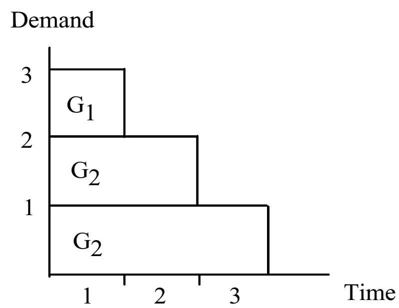

Figure 15: Inefficient dispatch for a sequential *1-horizontal* auction.

ator owns two plants with $K = 1$. The dispatch in Figure 15 is a result of equilibrium bids where G_2 submits a bid of $f_2 + 3v_2 - 1$ for *lot* 3, a bid that is below both her and G_3's cost to supply 3 MWh and wins *lot* 3. G_2 knows that by winning *lot 3*, she is committing herself to participate in only one of the two remaining auctions (due to capacity constraints). If all three generators were to participate in every auction, the upper bound on G_2's winning bid for *lot 2* is $f_1 + 2v_1 - 1$, and the upper bound on G_1's winning bid for *lot 1* is $f_2 + v_2 - 1$. Given G_2 has won *lot* 3, it is to G_1's advantage that G_2 win *lot* 2 and hence be removed from participating in the auction for *lot* 1. With G_2 no longer participating in the auction, G_1 is able to win *lot* 1 at a bid of $f_3 + v_3 - 1$. Therefore, G_1's optimal response is to not undercut G_2's bid for *lot 2* and allow G_2 to win *lot 2* with a bid of $f_3 + 2v_3 - 1$. Hence, G_2 is able to undercut G_3's lower costs for *lot 3* with the knowledge that it is in G_1's best interest to allow her to win *lot 2* at a earning a large profit margin. ■

Bundling demand so that there is one winner per lot did not remove the incentives for relatively inefficient generators to bid below cost nor prevent the resulting inefficient dispatch. It is the ability to change the upper bounds on winning bids via strategic interactions that allows an inefficient dispatch to be supported in equilibrium. As a consequence of the sequential nature of the auction and the lack of competition from identical generators, G_1 knows that, given G_2 wins both *lot* 3 and *lot* 2, the upper bound on a winning bid for *lot* 1 is raised from $f_2 + v_2$ to $f_3 + v_3$. Similarly, the sole reason winning *lot 3* is part of an overall profitable strategy for G_2 is that G_2 is able to change the upper bound on her winning bid for *lot* 2 from $f_1 + v_1 - 1$ to $f_3 + v_3 - 1$. If, instead, there were to exist several generators identical to G_1 or G_2, then this strategy is no longer optimal in equilibrium. If there exists another generator identical to G_2, G_2 will be unable to win *lot 2* at any price higher than epsilon above its cost, $f_2 + 2v_2$. For if she were to bid above her cost for *lot 2*, the identical generator would have the incentive to undercut its bid, and G_2 would win only *lot 3* at a loss. A similar argument can be made to show that, if there were to exist another generator identical to G_1, the upper bound on G_1's winning bid for *lot 1* remains $f_1 + v_1$, regardless of the outcomes of the two previous auctions. G_1 will undercut any bid for *lot 2* that is above its own cost of $f_1 + 2v_1$. Therefore, when there are many generators of the same type, it does not behoove G_2 (or any other generator) to bid below its cost for any lot, and the ability to support inefficient dispatches in equilibrium disappears.

Theorem 3 *In a complete information framework with many generators, a sequential* 1-horizontal *auction guarantees the efficient dispatch in equilibrium.*

Proof. The proof to Theorem 3 is identical to that of Theorem 4 and will be presented in the section 4.2.

4.2 1-Horizontal Simultaneous Auctions

This section concludes our search for an auction which guarantees efficiency in equilibrium.

Theorem 4 *In a complete information framework, a 1-horizontal simultaneous auction guarantees the efficient dispatch in equilibrium.*

Proof. In order to prove Theorem 4 it must be shown that (Step 1) the efficient dispatch can be supported in equilibrium and (Step 2) there do not exist any inefficient dispatches in equilibrium. For expository ease and without loss of generality, I will assume a daily demand as in Figure 16 and that there exist $M \geq 1$ generators of type i, denoted by $G_{ij}, i = 1...n, j = 1...M$, who each own one plant with $K = 2$.

Assume there exists sufficient capacity of each technology type i to satisfy the demand for which it is efficient. The proof to Theorem 4 holds in the case of a *1-horizontal* simultaneous auction when there are both a small number of generators of each type (e.g. there exists one generator of each

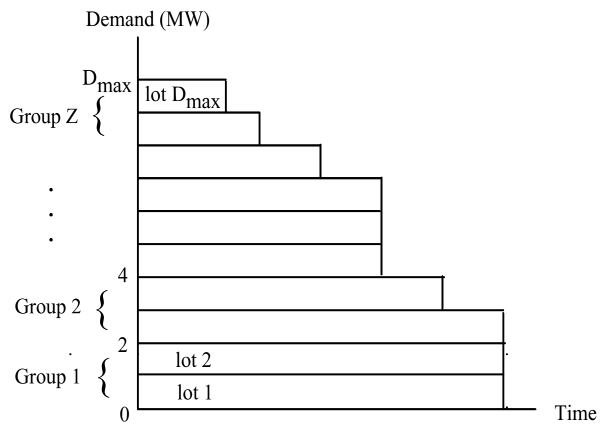

Figure 16: Daily demand for 1-horizontal auction : shown with lots and group partitioning.

type who owns a greater number of plants than is needed in the efficient dispatch, as was the case in section 4.1) and a large number of generators of each type (i.e., there are more generators of each type than demand for which it is the most efficient generator). {*Note*: For Theorem 3, we need to assume a large number of generators of each type.}

(Step 1) It is fairly simple to check that the following bids constitute an efficient equilibrium for a *1-horizontal* simultaneous auctions. Without loss of generality, assume that $D_{\max}$ is a multiple of K, i.e., $\frac{D\max}{2} \equiv Z$ is an integer.

For $i = 1...n,\ j = 2...M$, $\phi = 0...Z-1$, and $\pi = 2\phi + 1...2(\phi + 1)$

- G_{i1} bids $\frac{C_i(\text{total number of MWh in Group } \phi+1)}{\text{total number of MWh in Group } \phi+1} * (\text{MWh in lot } \pi)$ for lot π
- G_{ij} bids $\frac{C_i(\text{total number of MWh in Group } \phi+1)}{\text{total number of MWh in Group } \phi+1} * (\text{MWh in lot } \pi) + 1$ for lot π

Given the strict concavity of costs, in the efficient dispatch generators will win adjacent slices until their capacity is exhausted. Therefore, all the demand in Group ϕ will be supplied by the same generator, for $\phi = 1...Z$. The above bids capture this structure of the efficient dispatch by having generators bid on lots as functions of their cost of supplying the entire group. Given these bids, the least-cost supplier of Group ϕ will win all the lots in the group and the result is the efficient dispatch.

(Step 2) Denote the efficient dispatch by D^* and the total cost associated with the efficient dispatch by $C(D^*)$. Suppose that there *does* exist an inefficient dispatch, D, in equilibrium. Define B to be the sum of the winning bids and $C(D)$ to be the total cost associated with the inefficient dispatch D. In equilibrium, each generator must be making a non-negative profit. Given that each generator is paid what it bids and that all bids are submitted simultaneously[13], no generator has the incentive to bid below its cost. Summing over these constraints for the generators in dispatch D gives us,

$$C(D) \leq B \tag{1}$$

For dispatch D and its associated bids to constitute an equilibrium, no generator must have an incentive to change its bid given its opponents' bids. In particular, it must be unprofitable for the generator types in the efficient dispatch to undercut the bids in dispatch D. Summing these constraints for the generator types and their dispatch in D^* yields,

$$B < C(D^*) \tag{2}$$

Combining with equation (1) and (2) yields,

$$C(D) < C(D^*)$$

which contradicts the assumption that D^* is the efficient (least-cost) dispatch. ■

The key to the success of a simultaneous *1-horizontal* auction or sequential *1-horizontal* auction (with many generators) is the following: Under discriminatory pricing, having a simultaneous auction or many generators of the same type creates a strict upper bound on winning bids. When a generator is paid what it bids and there is no possibility of changing the upper bound on a winning bid through strategic interaction, it will never bid below its cost for a lot. A generator that bids below cost for a lot and wins would always be better off by withdrawing its bid and earning zero profit.

[13] In the case of a sequential 1-horizontal auction, the existence of identical generators prevents a generator from bidding below its cost in equilibrium. See discussion at the end of the proof.

There exist alternative bundling forms which limit the number of winner per lot to one. For example, the auctioneer can auction each individual 1 MWh separately. In this case, the unique equilibrium dispatch is efficient (and the proof to Theorem 4 remains the same). However, the auctioneer will be increasing the complexity of the auction without gaining any benefits.

5 Conclusion

As auction based mechanisms for electricity dispatch are emerging in previously regulated electricity supply industries, it is imperative to understand the effect of auction rules and structure on efficiency. This paper addresses exactly this relationship by asking which auction structures are sufficient to guarantee that demand is satisfied in a least-cost manner. What makes this an interesting and challenging question is the existence of electricity industry-specific characteristics such as 1) the existence of start-up costs, 2) the desire of generators to supply several MWh of demand, 3) the existence of cost dependencies in supplying MWh over both time and quantity dimensions, and 4) the inability to store electricity.

Using a complete information framework, I found that an auction design that allows for more than one winner per demand lot in an environment with strictly concave costs will not be able to guarantee efficiency in equilibrium. In order to guarantee efficiency, the auction design must be such that there is only one winner per demand lot and generators submit bids for all demand lots simultaneously.

There are two necessary and important extensions to the work done in this paper. The first is to test the validity of this paper's results in an environment where generators may make mistakes in placing bids, i.e., test if the bidding strategies which led to inefficient dispatches for the various auction mechanisms constitute a trembling-hand perfect equilibrium. A second future direction of research is to extend this study into an incomplete information framework. Generators' knowledge of their opponents' costs is a plausible assumption for the United Kingdom, but is not as plausible an assumption for California, where generation companies such as Pacific Gas and Electric and Southern California Edison are independent investor-owned utilities that have not had to share complete cost information.

Appendix A

Initially, I had thought the restricted bid structure, i.e., one in which generators are restricted to submitting one bid per unit per demand lot, to be an additional culprit to inefficiency in uniform-price auctions. Since the average cost of generating 1 MW depends upon the total number of MW generated, restricting generators to submit one bid (per generating plant) regardless of quantity should open the door for inefficiencies to arise in equilibrium.

Alternatively, allowing generators to submit a bid which is contingent on quantity should help reduce the existence of inefficient equilibria. To test this hypothesis, I analyze the same uniform, vertical, simultaneous auction presented in sections 3.1, but change the bid structure such that generators are allowed to submit a step supply function. Similar examples for the other auction mechanisms in Figure 3 can easily be constructed.[14]

Two-part bid in a Uniform, Vertical, Simultaneous Auction Assume the same framework of generators and demand as in section 3.1, and that the generators costs are given by Figure 5. The bid structure is changed such that a generator is allowed to submit a two-part bid in each hour. Since each generator can only generate in increments of 1 MW and has a capacity of $K = 2$, a 2-part bid can sufficiently capture and reflect its cost structure. Generators submit a two-part bid which is of the form

$$\begin{Bmatrix} \text{minimum price to generate 1 MW during hour t} \\ \text{minimum price to generate 2 MW during hour t} \end{Bmatrix}$$

for each hour $t = 1, 2$. I found that the additional flexibility of a quantity-specific bid did not eliminate inefficient dispatches from the set of Nash Equilibria. For example, the following bid strategies support the same inefficient dispatch found in section 3.1 and constitute a Nash Equilibrium bidding strategy (asterisks follow winning bids):

Given the bids in Table 3, the least-cost dispatch is to accept G_{21}'s bids for 2 MW during both hours, G_{31}'s bid for 2 MWh in hour 1 and 1 MWh in hour 2, and G_{1y}'s bid for 1 MW during hour 1, for some $y \in [1, M]$. This sets the clearing prices paid per MW in hours 1 and 2 to be $f_1 + v_1$

[14] Interested readers can contact the author for such examples.

and $v_1 - 1$, respectively[15]. At these clearing prices, G_{21} and G_{31} earns a positive profit and G_1 earn a zero profit. G_{21} and G_{31} have successfully submitted sufficiently low bids so as to make it impossible for any other generator to profitably undercut them in either time hour for either quantity level.

Not only does a richer bid structure not preempt inefficient dispatching in equilibrium, in addition it poses the combinatorial optimization problem for the auctioneer of identifying the least-cost manner of satisfying demand in each hour. In these simple examples, the generator could only generate at two possible output levels, while in reality the possible output levels are much larger.

[15] Note that G_{1y}'s bid for 1 MW is the highest accepted bid in hour 1 and G_{31}'s bid for 1 MW is the highest accepted bid for hour 2.

Generator	Bids for hour1	Bids for hour 2
$s = 2...M$, $y = 1...M$		
G_{1y}	$\binom{f_1+v_1*}{f_1+2v_1}$	$\binom{v_1}{2v_1}$
G_{21}	$\binom{0}{0*}$	$\binom{0}{0*}$
G_{2s}	$\binom{f_2+v_2}{f_2+2v_2}$	$\binom{f_2+v_2}{f_2+2v_2}$
G_{31}	$\binom{0}{0*}$	$\binom{v_1-1*}{2v_1-1}$
G_{3s}	$\binom{f_3+v_3}{f_3+2v_3}$	$\binom{f_3+v_3}{f_3+2v_3}$
G_{4y}	$\binom{f_4+v_4}{f_4+2v_4}$	$\binom{f_4+v_4}{f_4+2v_4}$

Table 3: Inefficient equilibrium 2-part bidding strategies

Appendix B

The following subgame perfect Nash equilibrium result in an inefficient dispatch in equilibrium for a discriminatory, horizontal, sequential auction. Assume that all generators submit the same bids for both of their plants unless otherwise specified. Once a particular plant has won a lot, it can no longer participate in later auctions.

$G1$**'s strategy**:

Bid for $f_3 + 3v_3 + 1$ for *lot 3*

- If I win *lot 3*, bid $f_3 + 2v_3 - 1$ for *lot 2*
  - If I win *lot 2*, bid ∞ for *lot 1*
  - If 2 wins *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
- If 2 wins *lot 3*, bid $f_3 + 2v_3 + 1$ for *lot 2*
  - If I win *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
  - If 2 wins *lot 2*, bid $f_3 + v_3 - 1$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
- If 3 wins *lot 3*, bid $f_1 + 2v_1$ for *lot 2*
  - If I win *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
  - If 2 wins *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - 1$ for *lot 1*

$G2$**'s strategy:**

Bid for $f_3 + 3v_3 - 1$ for *lot 3* for plant 1

Bid for $f_3 + 3v_3$ for *lot 3* for plant 2

If 1 wins *lot 3*, bid $f_3 + 2v_3 + 1$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3 - 1$ for *lot 1*

If I win *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 3*, bid $f_3 + 2v_3 - 1$ for *lot 2*

If 1 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 2*, bid ∞ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 3*, bid $f_3 + 2v_3 - 1$ for *lot 2*

If 1 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

$G3$'s strategy:

Bid for $f_3 + 3v_3$ for *lot 3*

If 1 wins *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

The result of these strategies is that $G2$ wins *lot 3* and *lot 2*, and $G1$ wins *lot 1*.

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